COMPUTATION OF BASE FLOW CHARACTERISTICS WITH UNIFORM BLOWING

I. S. Belotserkovets and V. I. Timoshenko UDC 533.6.011.6

Blowing at bluff body base was considered under different conditions and for small amount of blowing this problem was solved using dividing streamline model [1]. The effect of supersonic blowing on the flow characteristics of the external supersonic stream was studied in [2-4]. The procedure and results of the solution to the problem of subsonic blowing of a homogeneous fluid at the base of a body in supersonic flow are discussed in this paper. Analysis of experimental results (see, e.g., [5]) shows that within a certain range of blowing rate the pressure distribution along the viscous region differs very little from the pressure in the free stream ahead of the base section. In this range the flow in the blown subsonic jet and in the mixing zones can be described approximately by slender channel flow. This approximation is used in the computation of nozzle flows with smooth wall inclination [6, 7]. On the other hand, boundary layer equations are used to compute separated stationary flows with developed recirculation regions [8] in order to describe the flow at the throat of the wake. The presence of blowing has significant effect on the flow structure in the base region. An increasing blowing rate reduces the size of the recirculation region [9] and increases base pressure. This leads to a widening of the flow region at the throat, usually described by boundary-layer approximations. At a certain blowing rate the recirculation region completely disappears which makes it possible to use boundary-layer equations to describe the flow in the entire viscous region in the immediate neighborhood of the base section.

1. Consider two-dimensional flow developed with two interacting supersonic flows past the base from which a finite amount of subsonic fluid is blown. The fluids in the supersoniv streams and the subsonic jet are assumed perfect and homogeneous with constant specific heats. The stagnation temperatures in the streams and the jet could in general be different. The flow in the neighborhood of the base is, as usual, divided into inviscid external supersonic flow regions and "viscous" flow region which includes the potential core in the blown jet and mixing regions which become near wake viscous flow region. The effect of viscosity on the inviscid flow characteristics is taken into consideration through the inclusion of displacement thickness for the effective body [10]. The idealized picture for the given flow is shown in Fig. 1, where I and II are inviscid flow regions, III is the "viscous" flow region, y_1 and y_2 are asymptotic boundaries of viscous regions, y_1^* and y_2^* are boundaries of effective displacement body. Inviscid external flow characteristics are assumed known in the section A_1A_2. The gasdynamic parameters in the equivalent inviscid flows are found by integrating Euler equations. The flow in the "viscous" region is described through boundary-layer equations. In cylindrical coordinates these equations in nondimensional variables

$$\bar{x} = x/R_*,\quad \bar{y} = y/R_*,\quad \bar{u} = u/U_*,\quad \bar{v} = v/U_*,\quad \bar{\rho} = \rho/\rho_*,\quad \bar{H} = H/U_*^2,\ \bar{T} = T/T_*,\quad \bar{p}_e = p_e/(\rho_* U_*^2),\quad \bar{\mu} = \mu/\mu_*,\quad \mathrm{Re}_* = \rho_* U_* R_*/\mu_*,$$

are written in the following form with the addition of equations of state and a relation for viscosity

$$\frac{\partial}{\partial x}(y^j \rho u) + \frac{\partial}{\partial y}(y^j \rho v) = 0; \tag{1.1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp_e}{dx} + \frac{1}{\mathrm{Re}_* y^j}\frac{\partial}{\partial y}\left[y^j \mu \frac{\partial u}{\partial y}\right]; \tag{1.2}$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{1}{\mathrm{Re}_* y^j}\left\{\frac{\partial}{\partial y}\left[y^j \frac{\mu}{\mathrm{Pr}} \frac{\partial H}{\partial y}\right] + \mu\left(1 - \frac{1}{\mathrm{Pr}}\right) y^j \left(\frac{\partial u}{\partial y}\right)^2\right\}; \tag{1.3}$$

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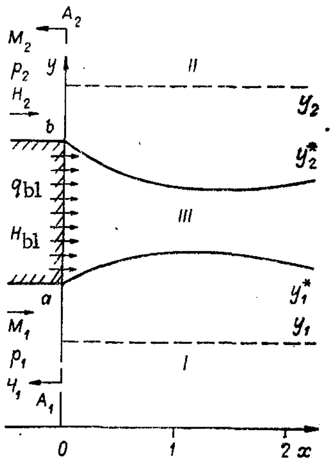

Fig. 1

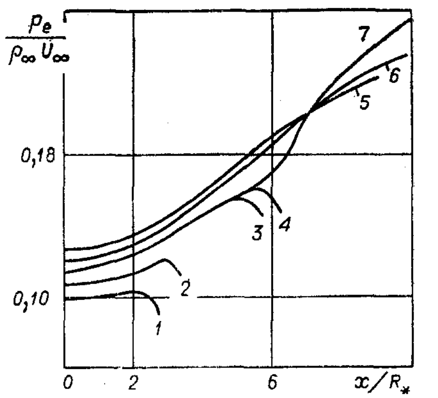

Fig. 2

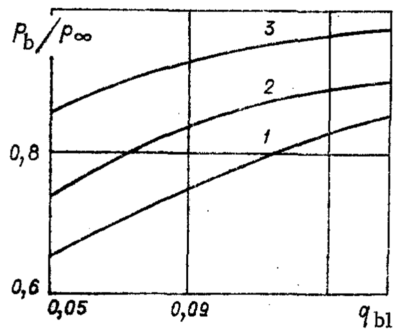

Fig. 3

$$p_e = \frac{\gamma}{\gamma - 1} \rho h, \tag{1.4}$$

where j = 0 corresponds to the planar case and j = 1 to the axisymmetric case. The following nomenclature is used: u, v, ρ, p_e, h, and μ are the streamwise and transverse velocity components, density, pressure, static enthalpy, and the coefficient of viscosity, respectively; $H = u^2/2 + h$; $\gamma = c_p/c_V$ is a constant; Pr is the Prandtl number; Re is Reynolds number. The bars over nondimensional parameters have been dropped in writing the system of Eqs. (1.1)-(1.4); the index * refers to the reference parameters used in nondimensionalizing and the index e indicates the value of the parameters at the edge of the equivalent inviscid flows.

The system of equations (1.1)-(1.4) is solved with the following boundary conditions:

$$u = u_{ek}, \ H = H_{ek} \ \text{ for } \ y \to y_k, \ k = 1, 2;$$

k = 1 corresponds to the boundary condition at the inner boundary of the "viscous" region and k = 2 indicates outer boundary. For symmetric "viscous" region boundary conditions at the inner boundary are written in the form

$$\partial u/\partial y = \partial H/\partial y = v = 0, \ y = y_1 \equiv 0.$$

In the general unsymmetric "viscous" region case, the boundary condition for transverse velocity component appreciably depends on the choice of the effective body, matching solutions at the boundaries, and will be discussed below.

Initial conditions for the system of equations (1.1)-(1.4) are specified at the base section in the form

$$u(0, y) = u_0(y), \ H(0, y) = H_0(y). \tag{1.5}$$

Equations (1.5) take into account nonuniformity of parameters in the jet as well as the presence of the initial boundary layer. The pressure distribution $p_e(x)$ is determined by solving the interaction problem of the "viscous" region with the external equivalent inviscid flows. The boundaries of the displacement body is determined as follows:

$$y_k^{*j+1} = y_{mk}^{j+1} + (1 + j) \int\limits_{y_{mk}}^{y_k} \left(1 - \frac{\rho u}{\rho_{ek} u_{ek}}\right) y^j dy, \quad k = 1, 2, \tag{1.6}$$

where y_{mk} is the streamline of constant mass dividing the fluid mass entering the base region because of blowing from the total mass occupied by the "viscous" region. In order to eliminate the unknown y_{mk} to determine the displacement boundary (1.6), the continuity equation is integrated across the viscous region with the limits $[y_{mk}, y_k]$:

$$\frac{1}{\rho_{ek} u_{ek}} \frac{d}{dx} \int\limits_{y_{mk}}^{y_k} \rho u y^j dy = y_k^j \left(\frac{dy_k}{dx} - \operatorname{tg} \theta_k\right), \quad k = 1, 2, \quad \operatorname{tg} \theta = \frac{v}{u}. \tag{1.7}$$

Since $y_{mk}(x)$ in (1.7) is the streamline, $dy_{mk}/dx = \tan \theta_{mk}$ along it.

Combining (1.6) and (1.7), and making the slender channel approximation $\partial p/\partial y = 0$, we get a relation connecting the direction of the velocity vector of the equivalent inviscid flow with respect to the displacement boundary and the pressure gradient:

$$y_k^j \operatorname{tg} \theta_k = y_k^{*j} \frac{dy_k^*}{dx} - \frac{y_k^{j+1} - y_k^{*j+1}}{j+1} \frac{M_{ek}^2 - 1}{M_{ek}^2} \frac{1}{\gamma p_e} \frac{dp_e}{dx}, \quad k = 1, 2. \tag{1.8}$$

The equation (1.8) is a system of equations for the unknown functions $\theta_1(x)$, $\theta_2(x)$, $p_e(x)$. In order to obtain the third, closure equation, Eq. (1.1) is transformed according to [11] to the form

$$\frac{\partial}{\partial y}(y^j \operatorname{tg}\theta) + \frac{1}{\gamma p_e}\frac{dp_e}{dx}\frac{M^2-1}{M^2} y^j + \frac{(\gamma-1)}{\gamma p_e \mathrm{Re}_*}\frac{1}{u}\left\{\frac{h}{u}\frac{\partial}{\partial y}\left[y^j \mu \frac{\partial u}{\partial y}\right]\right.$$
$$\left. - \frac{\partial}{\partial y}\left[y^j \frac{\mu}{\mathrm{Pr}}\frac{\partial h}{\partial y}\right] - y^j \mu \left(\frac{\partial u}{\partial y}\right)^2\right\} = 0, \quad M = \frac{u}{a} \tag{1.9}$$

and Eq. (1.9) is integrated across the "viscous" region to get

$$\gamma p_e (y_2^j \operatorname{tg}\theta_2 - y_1^j \operatorname{tg}\theta_1) + B\frac{dp_e}{dx} + A = 0. \tag{1.10}$$

A and B have the form

$$A = \frac{(\gamma-1)}{\gamma \mathrm{Re}_*}\int_{y_1}^{y_2} \frac{1}{u}\left\{\frac{h}{u}\frac{\partial}{\partial y}\left[y^j \mu\frac{\partial u}{\partial y}\right] - \frac{\partial}{\partial y}\left[y^j \frac{\mu}{\mathrm{Pr}}\frac{\partial h}{\partial y}\right] - y^j\mu\left(\frac{\partial u}{\partial y}\right)^2\right\} dy,$$

$$B = \int_{y_1}^{y_2} \frac{M^2-1}{M^2} y^j dy;$$

$\tan\theta_1$ and $\tan\theta_2$ are eliminated from (1.8) and (1.10) by writing the resulting equation for dp_e/dx:

$$\frac{dp_e}{dx} = \frac{\gamma p_e\left(y_2^{*j}\frac{dy_2^*}{dx} - y_1^{*j}\frac{dy_1^*}{dx}\right) + A}{\Delta}, \tag{1.11}$$

where

$$\Delta = \frac{1}{(1+j)}\left[\int_{y_1}^{y_2} \frac{\partial}{\partial y}\left(\frac{M^2-1}{M^2}\right) y^{j+1} dy - \frac{M_{e2}^2 - 1}{M_{e2}^2} y_2^{*j+1} + \frac{M_{e1}^2-1}{M_{e1}^2} y_1^{*j+1}\right].$$

Equation (1.11) does not depend on the choice of the asymptotic boundaries of the "viscous" region y_1 and y_2 since the derivatives of gasdynamic parameters in the viscous region approach zero as $y \to y_1, y_2$ [8, 10] in the present approximation.

Before formulating the viscous-inviscid interaction problem, let us consider in greater detail the specification of boundary conditions for the transverse velocity component at $y = y_1 \neq 0$. In this case the velocity component v is determined from Eq. (1.8) for k = 1 and $v = u \tan\theta$ for the given values of y_1, y_1^*, and dp_e/dx. The variation in v across the "viscous" region is found by integrating the first order Eq. (1.1). The condition for the transverse velocity component derived from Eq. (1.8) at k = 2 will be fulfilled automatically for dp_e/dx determined from Eq. (1.11).

Equation (1.11) can be considered a differential equation in the unknown functions p_e, y_1^*, and y_2^*. The remaining two equations are obtained by the determination of the flow in the inviscid, equivalent supersonic flows. In the general case, the solution to Euler equations results in the functional relation:

$$f_k\left(p_e, \frac{dy_k^*}{dx}\right) = 0, \quad k = 1, 2. \tag{1.12}$$

In the case when the external supersonic flow is described by the solution for a simple wave, functional relations (1.12) have the form of Prandtl-Meyer equations:

$$\left[1+\left(\frac{dy_k^*}{dx}\right)^2\right]^{-1}\frac{d^2y_k^*}{dx^2}=(-1)^k\frac{\sqrt{M_{ek}^2-1}}{M_{ek}^2}\frac{1}{\gamma p_e}\frac{dp_e}{dx},\quad k=1,2. \tag{1.13}$$

Thus, the system of three differential equations (1.11) and (1.12) or (1.11) and (1.13) for the case of a simple wave completely determines the interacting flow. This system will in the future be referred to as the system of viscous-inviscid interaction equations.

The system of viscous-inviscid interaction equations has a singularity associated with the determinant becoming zero at some downstream location. Numerical investigation of the singularity within a wide range of parameters shows that this point corresponds to a "saddle" point type of singularity. Analogous singularities arise in separated flow problems with developed recirculation regions, encountered in solving boundary layer equations using integral methods [8] and also in nozzle flows, e.g., [7, 12]. The generation of singularities is described in detail in these studies and it can be used in the present investigation.

2. As an example for the application of the above-described method to compute viscous-inviscid interaction and to study the effect of finite strength blowing on the base flow characteristics we considered uniform supersonic flow past a semiinfinite flat plate of finite width R_*. Subsonic blowing was introduced through the trailing edge of the plate with sufficient strength to prevent the formation of recirculation flows in the "viscous" region. The study was limited in the investigation of the region defined by the separation of external flow from the trailing edge and zero initial boundary-layer thickness. The distribution of parameters in the external inviscid flows was fully determined by specifying Mach numbers M_1 and M_2, static pressures p_1 and p_2, and total enthalpy H_1 and H_2 at the cross section. Inviscid equivalent flows past the effective body were described by Prandtl–Meyer equations for a simple wave. The gas in the external flow and in the jet was assumed perfect with $\gamma = 1.4$ and a Prandtl number Pr = 0.72. The coefficient of viscosity was determined by the relation $\mu/\mu_* = (T/T_*)^{\omega}$ where the index ω was assumed 0.5. Initial conditions for the system of Eqs. (1.1)-(1.4) were specified at the trailing edge section in the form of a parametric family in terms of the base pressure $p_b = p_e\ (x = 0)$:

$$u_0=\begin{cases}\dfrac{M_1}{M_2}\left(\dfrac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\dfrac{H_1}{H_2}\right)^{\frac{1}{2}}\times\\ \times\left[1+\dfrac{2}{(\gamma-1)M_1^2}\left(1-\left(\dfrac{p_b}{p_1}\right)^{1-\frac{1}{\gamma}}\right)\right]^{\frac{1}{2}}, \\ \left[\left(\dfrac{\gamma}{\gamma-1}\dfrac{p_b}{q_{bl}}\right)^2+2H_{bl}\right]^{\frac{1}{2}}-\dfrac{\gamma}{\gamma-1}\dfrac{p_b}{q_{bl}}, \\ \left[1+\dfrac{2}{(\gamma-1)M_2^2}\left(1-\left(\dfrac{p_b}{p_2}\right)^{1-\frac{1}{\gamma}}\right)\right]^{\frac{1}{2}},\end{cases}\qquad H_0=\begin{cases}H_1, & y<a,\\ H_{bl}, & a\leqslant y\leqslant b,\\ H_2, & y>b.\end{cases}$$

Here a and b are the ordinates of the trailing edge section $b = a + R_*$; q_{bl} the blowing strength defined by $q_{bl} = Q/\rho_* U_*^2 R_*$, where Q is the mass of blown gas; H_{bl} is the total enthalpy of the blown jet. Boundary-layer equations describing the flow in the "viscous" region were integrated numerically using four-point implicit difference scheme [13] and the viscous-inviscid interaction Eqs. (1.11) and (1.13) were integrated using Euler scheme [14]. The algorithm for computing the flow consisted of the determination of the singular integral curve of the system of viscous-inviscid interaction equations. Since the flow characteristics in the "viscous" region continuously depend on the initial conditions, the choice of one or the other integral curve is completely determined by the value of the free parameter p_b. The procedure for the determination of the singular integral curve is accomplished by the choice of this parameter.

Computed results are shown in Figs. 2-5. In all the computations the step size for the integration of boundary-layer equations was 0.025 and the number of computational grid points

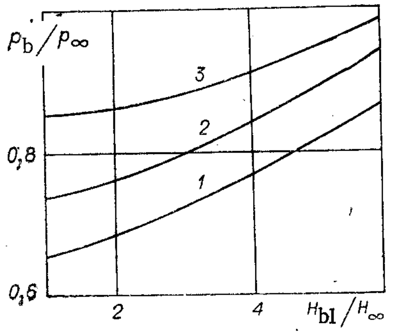

Fig. 4

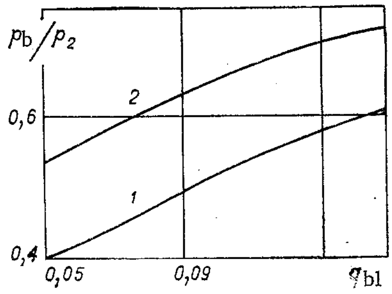

Fig. 5

in the transverse direction was 51. A typical picture of the behavior of integral curves in the neighborhood of the singularity in symmetric isoenergic flow past a flat plate is shown in Fig. 2 where the curves 1-7 correspond to values of p_b/p_∞ = 0.55; 0.60; 0.65; 0.65078; 0.70; 0.675; and 0.65156. The following were the values of the characteristic flow parameters: $a = -1$, $b = 1$, $M_1 = M_2 = M_\infty = 2.0$, $H_1 = H_2 = H_{b1} = 0.5 + 1/((\gamma - 1)M_\infty^2)$, $p_1 = p_2 = 1/(\gamma M_\infty^2)$, $q_{b1} = 0.05$, $Re_\infty = 500$. Computed results for the effect of blowing strength on the base pressure at Re_∞ = 500; 1000; and 5000 (curves 1-3, respectively) at the same reference conditions are shown in Fig. 3. The effect of nonisoenergic flow due to the blowing of hot jet on the base pressure is shown in Fig. 4 for the same values of Re_∞. The flow is assumed to be symmetrical. It is seen from Figs. 3 and 4 that an increase in blowing and the temperature of the blown gas leads to an increase in base pressure at different Reynolds numbers. The effect of blowing strength in the case of an unsymmetric flow past the plate is shown in Fig. 5, where curves 1 and 2 are for Re_∞ = 500 and 1000, respectively. The reference quantities are as follows: $a = 0.5$, $b = 1$, $M_1 = 3.0$, $M_2 = 2.0$, $H_1 = H_2 = H_{b1} = 0.5 + ((\gamma - 1)M_2^2)^{-1}$, $p_1 = p_2 = 1/(\gamma M_2^2)$. As seen from Fig. 5, an increase in the velocity of one of the flows leads to an increase in gas mass entrained by the flow from the jet which results in a decrease in base pressure when compared to the symmetric flow case. The geometry of the effective displacement body corresponding to the above numerical example is shown in Fig. 1 for $q_{b1} = 0.05$ and $Re_\alpha = 10^3$.

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